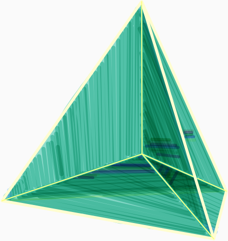

Analysis or Geometry? Geometric Analysis!

Math PhD Days in Pisa

February 5, 2026

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Università di Pisa

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Geometric Analysis

Geometric analysis is a field of mathematics at the interface of differential geometry and mathematical analysis. It draws inspiration from problems in topology, Riemannian/Lorentzian geometry, utilizing techniques from PDEs, the Calculus of Variations, and Geometric Measure Theory.

Objectives:

Solving geometric and topological problems using analytical techniques (ODEs, PDEs, GMT):

- Every compact, simply connected 3-manifold is diffeomorphic to the sphere $\mathbb{S}^3$ — **Poincaré Conjecture**.
- Every compact 3-manifold admits infinitely many immersed closed minimal surfaces — **Yau's Conjecture**.
- A complete, connected, non-compact Ricci-pinched 3-manifold is flat — **Hamilton's Conjecture**.

Geometric Analysis

Gaining deeper insights into PDEs through the geometric properties of their solutions:

- The Ricci flow with initial datum a surface with a finite number of conical points is well-posed.
- The boundaries of minimal clusters in the plane consist of circular arcs meeting at triple junctions forming 120° angles.

- Almost every level set of the inverse mean curvature flow is a curvature varifold.

Themes:

* Minimal Surfaces
* Geometric functionals involving curvature terms (e.g., Willmore energy)
* Geometric Flows (Ricci flow, mean curvature flow, inverse mean curvature flow)

Minimal Surfaces

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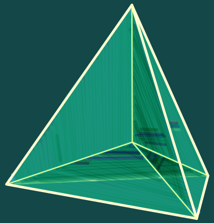
A surface $\Sigma \subset \mathbb{R}^3$ is a minimal surface if it is a critical point of the area functional. Equivalently, its mean curvature vanishes everywhere ($H = 0$).

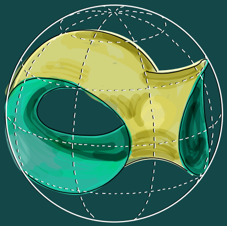
Topological realization of minimal surfaces:

- **Closed surfaces:** every closed orientable surface can be minimally embedded in $\mathbb{S}^3$ (Lawson, 1970).
- **Surfaces with boundary:** every compact orientable surface with boundary can be embedded in $\mathbb{B}^3$ as a free boundary minimal surface (Karpukhin, Kusner, McGrath, Stern, 2024).

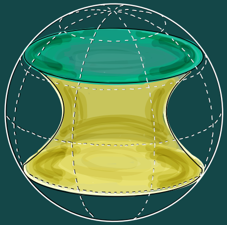
Plateau's Problem: given a closed curve $\gamma \subset \mathbb{R}^3$, find a surface Σ that minimizes the area functional among all surfaces bounded by γ.



Minimal cluster on the 1-skeleton of the tetrahedron



Free boundary minimal surfaces in $\mathbb{B}^3$ (genus 0, 3 boundary compontents)



Free boundary minimal surfaces in $\mathbb{B}^3$ (genus 0, 2 boundary components)

Geometric Flows

Geometric flow is an umbrella term for a class of geometric evolution equations, divided into two main families: intrinsic and extrinsic flows.

- Intrinsic flows involve the evolution of a Riemannian metric on a manifold without reference to an ambient space.
- Extrinsic flows describe the evolution of a hypersurface (or submanifold) within a Riemannian manifold. Typically, the velocity is related to the curvature of the hypersurface.

Flow	Introduced by	Applications / Context
Ricci flow	Hamilton '82	Poincaré Conjecture Thurston Geometrization
Mean Curvature	Huisken '84	Singularity Analysis
Inverse Mean Curvature	Huisken-Ilmanen '01	Riemannian Penrose Inequality
Surface Diffusion	Mullins '57	Thermal grooving models
Network flow	Mullins '56	Grain growth in metals
Willmore flow	Kuwert-Schätzle '02	Elastic energy

Let N be a smooth manifold and $F_0 : N \to \mathbb{R}^{n+1}$ a smooth immersion. Denote by H the mean curvature and by ν the inward unit normal.

Mean Curvature Flow

$$\begin{cases} \partial_t F = \mathrm{H}\nu \\ F(\cdot,0) = F_0 \end{cases} \qquad \text{(MCF)}$$

- Shrinking flow
- L^2-gradient flow of the Area functional
- Quasilinear second order parabolic PDE

Inverse Mean Curvature Flow

$$\begin{cases} \partial_t F = -\frac{1}{\mathrm{H}}\nu \\ F(\cdot,0) = F_0 \end{cases} \qquad \text{(IMCF)}$$

- Expanding flow
- No simple variational structure
- Reformulation with a degenerate elliptic PDE

MCF and IMCF

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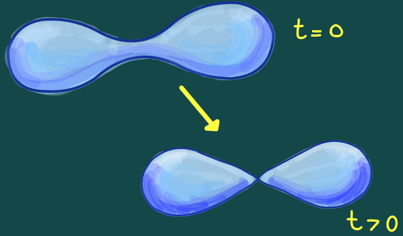

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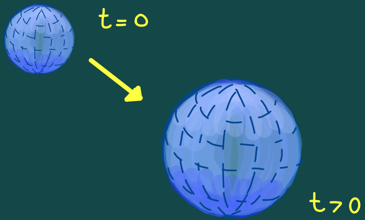

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Key Objectives

- Well-posedness, regularity, and singularity analysis.
- Monotonicity formulas and geometric inequalities.

* Joint UNIPI-SNS Geometric Analysis Seminar

* Reading Seminars
  - Allen-Cahn and Yang-Mills
  - Einstein metrics

* Conferences at "Centro De Giorgi"
  - Topics in Geometric Analysis
  - Morphogenesis and morphing 200 years after Gauss
  - Local and non-local methods in geometric analysis
  - The Pisan Workshop Saga

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Activities

(#)

- Joint UNIPI-SNS Geometric Analysis Seminar

- Reading Seminars
  - Allen-Cahn and Yang-Mills
  - Einstein metrics

(★)

- Conferences at “Centro De Giorgi”
  - Topics in Geometric Analysis (✱)
  - Morphogenesis and morphing 200 years after Gauss
  - Local and non-local methods in geometric analysis (#)
  - The Pisan Workshop Saga (★)

(✱)

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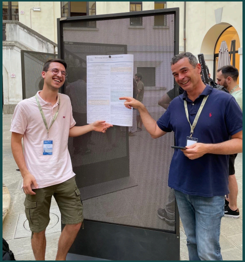
Group Members

Current PhD Students

Francesco Malizia (SNS)



Giorgio Gatti (Padova)

Current PostDocs

Andrea Bisterzo, Albachiara Cogo, Liangjun Weng

PhD Students and PostDocs of the past five years

Michele Caselli, Antonia Diana, Mattia Freguglia, Luciano Sciaraffia,
Luca Benatti, Mattia Fogagnolo, Fritz Hiesmayr,
Francesco Palmurella, Francesca Tripaldi, and others...

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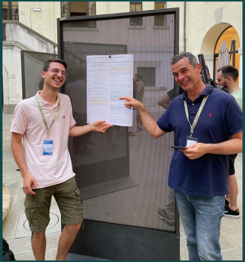

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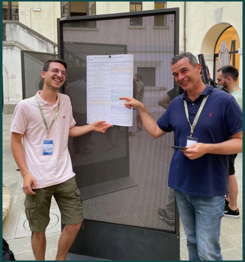

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G.d.H. M.F. M.C. F.H. A.M. L.S. C.A. L.W.

Giorgio Gatti - PhD student in Padova - co-supervised with Mattia Fogagnolo
Weak IMCF and applications

The aim of the thesis is to establish the existence of proper weak solutions to the IMCF in non-smooth spaces (Riemannian manifolds with C^0 metrics and RCD spaces). The existence of weak solutions will be pivotal in showing the validity of geometric inequalities such as Willmore and Minkowski-type inequalities, Geroch Monotonicity formula and Penrose-type inequalities.

Antonia Diana - PhD student at SSM-Napoli - co-supervised with Carlo Mantegazza
The Surface Diffusion Flow: Long–Time Behaviour and Asymptotic Stability

This thesis focuses on the global existence and stability of surface diffusion flow - the H^{-1}-gradient flow of the area functional, where the normal velocity is given by the Laplacian of the mean curvature - for smooth boundaries of subsets of the n-dimensional flat torus.

Julia Menzel - PhD student at Uni Regensburg - co-supervised with Harald Garcke
Boundary Value Problems for Evolutions of Willmore Type

The first part of the thesis is devoted to the Willmore flow of compact open hypersurfaces of dimension two or higher immersed in Euclidean space with Navier boundary conditions. The second part concerns the study of the long-time behavior of elastic flow of networks.

A.M. main research themes:

NEXT TALK!

A.P. main research themes:

- Evolution by mean curvature of **networks** and **clusters**.
- Analysis of **higher order geometric flows** (elastic flow, Willmore).
- Inverse Mean Curvature flow, p-harmonic functions and applications (geometric inequalities, applications to mathematical general relativity, topological applications).

Thank you for your attention!

Next Event

Late summer workshop in geometric analysis

31 August – 3 September 2026

Università di Padova

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